Exercise 38

For the following exercises, solve the equations over the complex numbers.

$$x(x-2)=10$$

Solution

Rewrite the equation in standard form.

$$x^2-2x=10$$

$$x^2-2x-10=0$$

The two terms with x, x^2 and $2x$, cannot be combined, so it's necessary to complete the square to solve for x. Recall the following algebraic identity.

$$(x+B)^2=x^2+2xB+B^2$$

Notice that $2B=-2$, which means $B=-1$ and $B^2=1$. Add and subtract 1 from the left side and apply the identity.

$$(x^2-2x+1)-10-1=0$$

$$(x+(-1))^2-11=0$$

$$(x-1)^2-11=0$$

Now that x appears in only one place, it can be solved for. Add 11 to both sides.

$$(x-1)^2=11$$

Take the square root of both sides.

$$\sqrt{(x-1)^2}=\sqrt{11}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x-1$.

$$|x-1|=\sqrt{11}$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x-1=\pm\sqrt{11}$$

Add 1 to both sides.

$$x=1\pm\sqrt{11}$$

Therefore, $x=\{1-\sqrt{11},1+\sqrt{11}\}$.